Comparison of the Failure Conditions for Creep, Stress Relaxation, and Constant Strain Rate Measurements to Predict Pipe Burst for Two ABS Materials Using the Universal Viscoelastic Model

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Received 31 March 2003; accepted 5 February 2004
DOI 10.1002/app.20377

ABSTRACT: The universal viscoelastic model was further validated in this study using two acrylonitrile–butadiene–styrene (ABS) viscoelastic materials to better elucidate the direct relationship between the failure criterion characteristics involving creep, constant strain rate, stress relaxation, and pipe burst. Using the yield strain as the failure criterion for constant strain rate and stress relaxation measurements and the strain at critical creep, the failure condition for creep, it was found that the universal viscoelastic model allowed these failure criteria to yield remarkably good agreement on a projected time scale. The relationship of the failure criteria between these three different techniques for characterizing a viscoelastic material was successfully used to identify several complementary approaches to predict long-term pipe burst for two different ABS materials. The pipe burst data for ABS-A appeared to fit the Tresca failure criterion better than the von Mises failure criterion, as predicted from failure criteria using constant strain rate, creep, and stress relaxation measurements. For ABS-A the extrapolated creep and the constant strain rate failure criterion appeared to best predict the Tresca failure criterion pipe burst data. However, the hoop burst stress for ABS-N, adjusted with the von Mises failure criterion modification, was found to give the best agreement with the yield stress equivalent failure criterion for constant strain rate, creep, and stress relaxation. For ABS-N, the extrapolated creep failure criteria appeared to best predict the von Mises pipe burst failure criterion. In this study, the relationships between the failure criteria for the three different experimental techniques of constant strain rate, creep, and stress relaxation were shown to be reasonably interchangeable relative to a three-dimensional failure configuration such as pipe burst. Because this interchangeability approach was found to work so well in the laboratory, there is no reason to believe that this same technique, using the universal viscoelastic model, would not also work as well to predict failure criteria for design applications using finite-element analysis. © 2004 Wiley Periodicals, Inc. J Appl Polym Sci 93: 247–260, 2004

Key words: viscoelastic properties; creep; stress; strain; modeling

INTRODUCTION

Recently a series of articles written by this author[1–5] characterized a new universal viscoelastic model that describes a definitive relationship between constant strain rate, creep, and stress relaxation analysis for viscoelastic polymeric compounds. This new universal viscoelastic model has been further validated using two acrylonitrile–butadiene–styrene (ABS) viscoelastic materials[6] to better elucidate the direct relationship between creep, constant strain rate, and stress relaxation measurements. The relationship of the failure criteria between these three different techniques for characterizing a viscoelastic material will be addressed in this article to identify several complementary approaches to predict long-term pipe burst for two different ABS materials.

In recent years the need for a simple analysis approach, which relates creep, stress relaxation, and constant strain rate measurements all in one simple model, has been generated as a result of the extended use of finite-element analysis involving polymeric compounds[7] and composites.[8] Before the introduction of this new universal viscoelastic model, several authors had attempted to describe two or more of these viscoelastic concepts in one unifying formulation.[9,10] However, most of the efforts over the years have been to simulate uniaxial creep,[11,12] stress relaxation,[9] or constant strain rate data[13–16] separately. This new formulation approach offers a reasonably simple process by which one can shift from a constant strain rate configuration to a creep calculation or stress relaxation configuration without changing formulation considerations or without stress or strain discontinuities.

Because it has previously been very difficult to predict a direct relationship between constant strain rate, creep, and stress relaxation measurements, failure conditions with three dimensions of stress have been even much more difficult to predict. For example, the prediction of failure in pressurized plastic pipe and

Journal of Applied Polymer Science, Vol. 93, 247–260 (2004)

aerosol bottles has been an area of significant practical interest. The early work of Baer et al.[17] was able to successfully show that the failure of pressurized plastic pipe as a function of time could be predicted using stress relaxation measurements at the yield strain. Over the years several modifications and improvements have been generated in the literature[18–25] that have extended the concepts initiated in Baer's early work regarding the prediction of plastic pipe failure.

The three classical approaches, for prediction of a failure stress in three-dimensional space, were originally developed primarily for metals. These three classical approaches, to evaluate three-dimensional failure stress primarily for metals, were generated by Coulomb[26] (1773), Tresca[27] (1864, 1867), and von Mises[28] (1913). Several modifications of each of these failure conditions have been addressed extensively in reviews by Ward[29] and Thorkildsen.[30] Baer et al.[17] found that the original von Mises failure conditions nicely fit their polyethylene pipe burst data.

This study then will show that constant strain rate, creep, and stress relaxation measurements evaluated using the universal viscoelastic model all predicted essentially the same pipe burst failure conditions for the two ABS materials evaluated. The pipe burst failure criterion were assumed to be predicted by either the Tresca or the von Mises failure conditions for these two ABS materials.

APPLICATION OF THE TRESCA AND VON MISES FAILURE CONDITIONS TO THE PREDICTION OF PIPE BURST

The von Mises failure condition[28–30] for three-dimensional stress failure in terms of the principal stresses, σ_1, σ_2, and σ_3 can be written as

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2 \quad (1)$$

This result can easily be obtained if the failure condition is assumed to occur at the yield stress for uniaxial tension where $\sigma_1 = \sigma_y$, $\sigma_2 = 0$, and $\sigma_3 = 0$.

For a pressurized pipe, with P = pressure, R = radius, and a negligible thickness t, then

$$\sigma_1 = \sigma_H = \frac{PR}{t} = \text{Hoop Stress} \quad (2)$$

$$\sigma_2 = \sigma_L = \frac{PR}{2t} = \text{Longitudinal Stress} \quad (3)$$

and $\sigma_3 \cong 0$.

Thus $\sigma_2 = \sigma_L = (\sigma_H/2)$ and substituting into eq. (1) gives

$$\sigma_y = \left(\frac{\sqrt{3}}{2}\right)\sigma_H \quad (4)$$

Given that the failure stress in pipe burst is normally the hoop stress σ_H, then the equivalent uniaxial yield stress for the von Mises failure criterion would be obtained by multiplying the hoop stress by the factor $\sqrt{3}/2$, as indicated in eq. (4).

The Tresca condition[27,29,30] for failure assumed that failure occurred when the maximum shear stress reaches a critical value, that is,

$$\sigma_1 - \sigma_3 = \text{Constant} \quad (5)$$

with

$$\sigma_1 > \sigma_2 > \sigma_3$$

For uniaxial tension failure at the yield stress, $\sigma_1 = \sigma_y$, $\sigma_2 = 0$, and $\sigma_3 = 0$. Substituting into eq. (5) gives

$$\sigma_1 - \sigma_3 = \sigma_y - 0 = \sigma_y = \text{Constant} \quad (6)$$

Thus for pipe burst, if $\sigma_1 = \sigma_H$, $\sigma_2 = \sigma_L = (\sigma_H/2)$, and $\sigma_3 = 0$, then substituting into eq. (6) gives the Tresca failure condition as

$$\sigma_y = \sigma_H \quad (7)$$

Both the Tresca and the von Mises conditions will be addressed for the pipe burst data generated in this study. However, before addressing the pipe burst data, the new universal viscoelastic model will be briefly reviewed to generate several approaches to evaluate the uniaxial failure condition often considered to be equivalent to yield stress σ_y as a function of time.

SUMMARY OF THE NEW UNIVERSAL VISCOELASTIC MODEL

The basic universal viscoelastic model can be characterized with the following equations as described in detail elsewhere[1–5]:

$$\frac{\sigma}{\sigma_y} = K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 \quad (8)$$

$$K = \frac{E}{\sigma_y} \quad (9)$$

$$A_2 = \frac{(3 - 2K\varepsilon_y)}{K^2\varepsilon_y^2} \quad (10)$$

$$A_3 = \frac{(K\varepsilon_y - 2)}{K^3\varepsilon_y^3} \quad (11)$$

$$\sigma_y = \frac{\beta}{t_y^n} \tag{12}$$

$$t = \frac{\varepsilon}{\dot{\varepsilon}_i} \tag{13}$$

$$\varepsilon_y = \varepsilon_\infty + \varepsilon_0(1 - e^{-\gamma\dot{\varepsilon}_i}) \tag{14}$$

where

$A_2, A_3, \ldots, A_i$ = variable constants for a series of strain rates for the same polymer formulation
E = elastic modulus, psi
K = ratio of modulus to the yield strength that is assumed to be a constant for all strain rates
n = efficiency of yield energy dissipation
t = time to achieve a strain ε, min
t_y = time to yield, min
β = energy dissipation constant
$\dot{\varepsilon}_i$ = characteristic strain rate
ε = characteristic strain
ε_y = yield strain
ε_∞ = long-term limiting strain to yield (when the strain rate approaches an infinitely small value or $\dot{\varepsilon}_i \rightarrow 0$)
ε_0 = short-term supplemental strain to yield limit (when the strain rate approaches an infinitely large value or $\dot{\varepsilon}_i \rightarrow \infty$)
γ = exponential strain rate constant for yield strain
σ = characteristic stress, psi
σ_y = engineering yield stress, psi

Combining eqs. (8)–(14) gives

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_y}\right)^n [K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{15}$$

It is also interesting to address the case that exists using eq. (14) at very low elongation rates $\dot{\varepsilon}_i$ or, equivalently, at very long times t. For this case note that the yield strain ε_y, described by eq. (14), approaches a limiting value ε_∞ as

$$\varepsilon_y \rightarrow \varepsilon_\infty \quad \text{as} \quad \dot{\varepsilon}_i \rightarrow 0 \text{ (very long times)}$$

For this case the constants A_2 and A_3 then approach the following values:

$$A_2' = \frac{(3 - 2K\varepsilon_\infty)}{K^2\varepsilon_\infty^2} \tag{16}$$

$$A_3' = \frac{(K\varepsilon_\infty - 2)}{K^3\varepsilon_\infty^3} \tag{17}$$

and eq. (15) then reduces to

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty}\right)^n [K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3] \tag{18}$$

Combining eqs. (13) and (18) then gives

$$\sigma = \beta\left(\frac{\varepsilon}{\varepsilon_\infty}\right)^n \left(\frac{1}{t^n}\right)[K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3] \tag{19}$$

Note that eqs. (18) and (19) apply only to the condition where the yield strain ε_y approaches its limiting value of ε_∞ as a result of the strain rate $\dot{\varepsilon}_i$ approaching zero. Equation (19) can also be rearranged for creep analysis in the following form:

$$t = \left(\frac{\varepsilon}{\varepsilon_\infty}\right)\left(\frac{\beta}{\sigma}\right)^{1/n} [K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3]^{1/n} \tag{20}$$

As was indicated in a previous publication,[1] eqs. (18), (19), and (20) can be extremely helpful when trying to address either creep or stress relaxation at very low strain rates $\dot{\varepsilon}_i$ or at very long times t. However, eqs. (8)–(14) can also be used to describe a complete series of uniaxial constant strain rate curves for a given polymer formulation and/or processing condition, as described in previous publications.[1–5]

This universal viscoelastic model also identified the estimated failure strain in creep that has been designated as the "critical creep strain" ε_{CC}, which can be obtained from eq. (20) by setting $(dt/d\varepsilon) = 0$ and solving for the resulting equation for the strain at critical creep ε_{CC}, to give

$$\varepsilon_{CC} = \left(\frac{-(n+2)A_2' \pm \sqrt{(n+2)^2A_2'^2 - 4(n+1)(n+3)A_3'}}{2(n+3)A_3'K}\right) \tag{21}$$

Also note that, when $n = 0$, then eq. (21) yields the limiting "critical creep strain" value of $\varepsilon_{CC} = \varepsilon_\infty$. Thus the greater the value of n, the greater the difference between the values of critical creep strain ε_{CC} and the limiting yield strain ε_∞.

By definition, a straight line for secondary creep would involve the following equation:

$$\varepsilon = \left(\frac{d\varepsilon}{dt}\right)t + \varepsilon_I \tag{22}$$

where $d\varepsilon/dt$ is the slope in secondary creep and ε_I is the intercept strain. In previous publications[4–6] it has been shown that the universal visoelastic model does yield eq. (22) with appropriate algebraic manipulations. Following this analysis, the slope $d\varepsilon/dt$ and the intercept strain ε_I, can be calculated as

$$\frac{d\varepsilon}{dt}=\frac{\varepsilon n}{t}\left(\frac{1+A_2'(K\varepsilon)+A_3'(K\varepsilon)^2}{1+n+(2+n)A_2'(K\varepsilon)+(3+n)A_3'(K\varepsilon)^2}\right) \tag{23}$$

$$\varepsilon_I=\varepsilon\left(\frac{1+2A_2'(K\varepsilon)+3A_3'(K\varepsilon)^2}{1+n+(2+n)A_2'(K\varepsilon)+(3+n)A_3'(K\varepsilon)^2}\right) \tag{24}$$

The average slope and intercept must be obtained by averaging, over a series of equally spaced data points, in the secondary slope region, such that

$$\left(\frac{d\varepsilon}{dt}\right)_{\mathrm{Ave}}=\frac{\sum_{i=1}^{i=k}\left(\frac{d\varepsilon}{dt}\right)_i}{k} \tag{25}$$

$$\varepsilon_{I\mathrm{Ave}}=\frac{\sum_{i=1}^{i=k}\varepsilon_{I_i}}{k} \tag{26}$$

It has also been shown that, for the same material, all the secondary creep straight lines must pass through the same average intercept creep strain $\varepsilon_{I\mathrm{Ave}}$, designated[14] as the "projected elastic limit strain."

Finally, the relationship between instantaneous extensional viscosity η_E, the creep stress σ, and the strain rate $d\varepsilon/dt$, during the creep process, can be defined as

$$\eta_E=\frac{\sigma}{\left(\frac{d\varepsilon}{dt}\right)} \tag{27}$$

Use of the universal viscoelastic model, as described elsewhere,[5,6] was then shown to yield an equation of the following form:

$$\eta_E=\lambda_E\left(\frac{d\varepsilon}{dt}\right)^{n-1} \tag{28}$$

where the extensional viscosity constant λ_E can be shown to be

$$\lambda_E=\left(\frac{\beta K\varepsilon}{\varepsilon_\infty^n}\right)\left(\frac{1}{n}\right)^n(1+n+(2+n)A_2'(K\varepsilon)+(3+n)A_3'(K\varepsilon)^2)^n(1+A_2'(K\varepsilon)+A_3'(K\varepsilon)^2)^{(1-n)} \tag{29}$$

Note that eq. (28) is very similar to the power law relationship that is so commonly used for shear viscosity as a function of the shear rate for a viscoelastic non-Newtonian fluid. It is also important to recognize that the value for the extensional viscosity constant λ_E, as described by eq. (29), yields essentially a constant when averaged over the strains involved in secondary creep, such that

$$\lambda_{E\mathrm{Ave}}=\frac{\sum_{i=1}^{i=k}\lambda_{E_i}}{k} \tag{30}$$

It is also clear, from eq. (29), that the magnitude of this viscosity constant is also strongly dependent on the efficiency of yield energy dissipation n, which was previously shown[2] to be primarily a measure of the viscoelastic character of a material. We will expand further on this important observation in the next sections of this article.

EXPERIMENTAL MATERIALS AND TESTING MEASUREMENT CONSIDERATIONS

The two acrylonitrile–butadiene–styrene (ABS) materials evaluated in this study, provided by the GE facility in Washington, WV, were designated as 25383-A (ABS-A) and LL-4102-N (ABS-N). All the test samples were prepared using a Brabender single-screw extruder, using a slit die that yielded a cross section that was approximately 0.75 in. wide with a thickness of approximately 0.0625 in. Because this thickness normally cannot be effectively evaluated using standard extensomers, it was decided to make tensile dumbbells with a gage length of approximately 20 in. to increase the accuracy and minimize any potential gage length error. The extruded strips were cut into tensile dogbone-shape specimens using a specially made cutting template on a TensilKut sample cutter. The final dogbone-shape specimen width in the gage length area was approximately 0.5 in. An Instron (Canton, MA) gear-driven testing machine was used to obtain the tensile measurements at three different strain rates (2, 0.2, and 0.02 in./min). The stress relaxation measurements, for the ABS materials evaluated in this study, were also evaluated on the Instron gear-driven testing machine.

The creep measurements were evaluated using a typical static tensile configuration with the lower grip load clamp capable of accepting standard scale weights to generate the tensile stress in the 20-in.-long dumbbell specimens. The extension movement for creep measurements were followed using a cathetometer with a travel length of approximately 4.5 ft., similar to the Model TC-II made by Titan (San Diego, CA). The creep results for ABS-A were evaluated at three different stresses (4138, 4635, and 5197 psi). Similarly, the creep results for ABS-N were also evaluated at three different stresses (3227, 3703, and 4322 psi).

TABLE I
Summary of Universal Viscoelastic Constants for Constant Strain Rate, Creep, and Stress Relaxation for Two ABS Materials

	ABS material					
	25383-A			LL-4102-N		
Property	Constant strain rate	Creep	Stress relaxation	Constant strain rate	Creep	Stress relaxation
Efficiency of yield energy dissipation, n	0.05227	0.05109	0.06036	0.04910	0.07031	0.06997
Beta, β, psi	6410.16	6352.73	6956.88	5110.74	5384.23	5144.64
Ratio modulus/yield strength, K	49.36			49.39		
Gamma, γ	73.47			95.31		
Alpha, α	-0.27			0.25		
Epsilon zero, ε_0	-0.00368			0.00262		
Epsilon infinity, ε_∞	0.03658			0.03317		
Strain at critical creep, ε_{CC}		0.03734			0.03399	
Elastic limit strain, ε_{EL}		0.02248			0.02204	
Extensional viscosity constant, λ_E, psi-min		7876.89			7350.73	

The technique used in this study to evaluate both the long-term pipe burst measurements was previously reported by Malpass.[10] The long-term hydrostatic pipe burst test used was consistent with the procedure described in ASTM D1598. Consistent with this procedure the time to failure was measured under continuous hydrostatic pressure of 18-in. lengths of 1-in. schedule-40 extruded pipe. All specimens were immersed in a circulating water tank controlled at 23 ± 1°C. Each specimen was allowed a minimum of 24 h at 23°C before being pressurized.

COMBINED SUMMARY OF THE UNIVERSAL VISCOELASTIC MODEL CONSTANTS FOR CONSTANT STRAIN RATE, CREEP, AND STRESS RELAXATION FOR ABS MATERIALS 25383-A AND LL-4102-N

The analysis of the constant strain rate, creep, and stress relaxation measurements for ABS materials 25383-A (ABS-A) and LL-4102-N (ABS-N), using the universal viscoelastic model, were described in some detail in a previous publication.[6] Consequently, a convenient summary of the resulting constants from these measurements using the universal viscoelastic model is tabulated in Table I. The definition of the constants, summarized in Table I, have been defined and characterized using the equations indicated in an earlier section of this article. As indicated in Table I the different measurement techniques appeared to yield nearly the same physical constants for each material. However, some of these constants, for the same measurement (like constant strain rate), were quite different for materials ABS-A and ABS-N. At this point it is useful to extract the failure criteria information from the three property measurements for each of the materials, summarized in Table I, to effectively predict pipe burst failure.

CONSTANT STRAIN RATE FAILURE CRITERIA FOR MATERIALS ABS-A AND ABS-N

As indicated in Table I, the constants for the short-term supplemental strain to yield limits ε_0 for these two ABS materials have exactly opposite signs. The value of ε_0 is negative for ABS-A and is positive for ABS-N. This means that the strain to yield for ABS-A, for constant strain rate measurements, decreases with an increase in strain rate but the strain to yield for ABS-N increases with an increase in strain rate. Brinson and DasGupta[9] point out that Crochet[31] predicted theoretically that the yield strain should decrease with an increase in strain rate. Brown[16] also predicted theoretically that the yield strain should decrease with an increase in strain rate. Previously this author[1] also found that the yield strain for polyethylene appears to decrease with an increase in strain rate similar to that found for ABS-A. However, the results for ABS-N appear to be consistent with the previous results found by Malpass[10] for another ABS material and by Brinson and DasGupta[9] for a polycarbonate. For these last three materials it was found that the strain to yield increased with an increase in the strain rate.

According to Brown,[16,32,33] Buchdahl,[14] and Robertson,[34] the ratio of the modulus to the yield strength $K = (E/\sigma_y)$ is normally a constant for a given polymer formulation that typically ranges from 40 to 60. As indicated in Table I, the average ratio of modulus to the yield strength K, from constant strain rate measurements, was approximately the same for both of these materials. In addition, the value of $K = 49.4$ for these two materials was in the middle of the range of 40–60 previously reported by Brown,[16,32,33] Buchdahl,[14] and Robertson[34] for this ratio.

Based on the universal viscoelastic model, the yield stress σ_y, or the failure criterion for constant strain rate

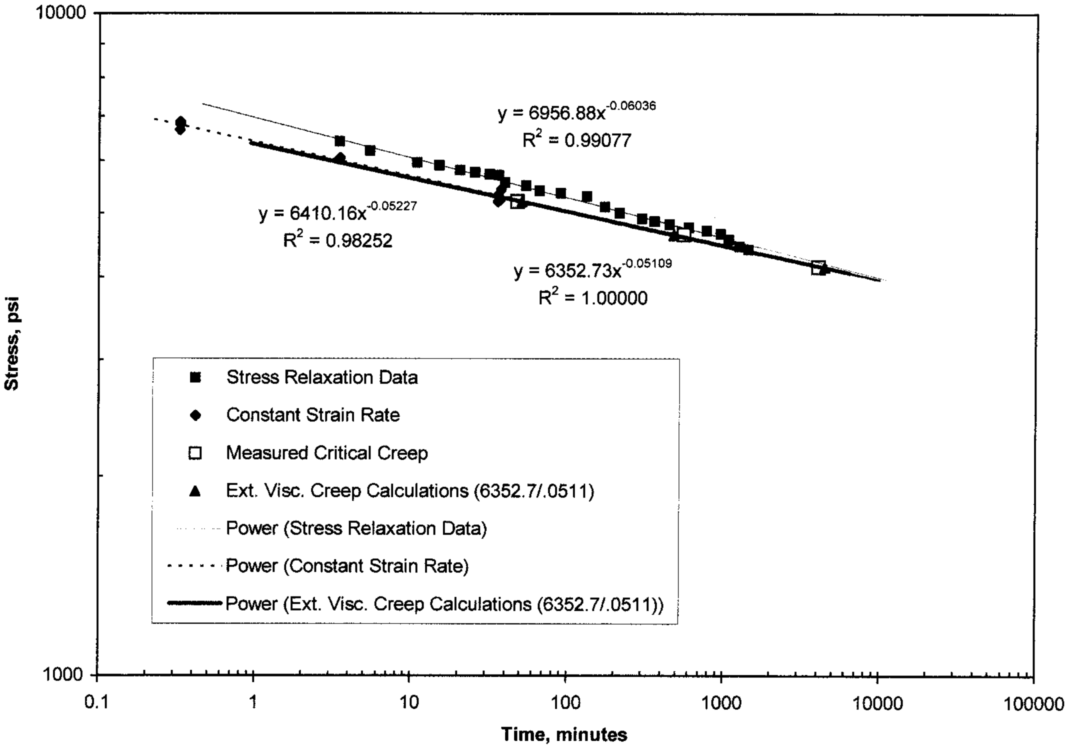

Figure 1. Constant strain rate, stress relaxation, and creep stress versus time for ABS material 25383-A.

data as a function of the time to yield t_y, can be described by the eq. (12):

$$\sigma_y = \frac{\beta}{t_y^n} \tag{12}$$

The relationship, described by eq. (12), between yield stress σ_y and time to yield t_y, is also currently included in ASTM D2837-98a (Standard Test Method for Obtaining Hydrostatic Design Basis for Thermoplastic Pipe Materials). In addition, Reinhart[18] used this relationship to predict long-term failure stress (which is normally close to the stress evaluated from the stress relaxation of the yield stress) as a function of time.

Although the efficiency of yield energy dissipation n, for both materials ABS-A and ABS-N, is close to the same value as indicated in Table I, the value of β is significantly different for these two materials. This suggests that material ABS-A should be able to survive at a higher stress than material ABS-N for a longer time. The actual failure criteria, or the yield stress versus the time to yield for the constant strain rate data used to generate the values of β and n in Table I, have been included in Figure 1 for ABS-A and in Figure 2 for ABS-N. It should also be recalled that the constants, from constant strain rate measurements, theoretically include all the constants needed to evaluate the universal viscoelastic model for any stress or strain condition. This means that, theoretically, these constants can also be used to predict constant strain rate, creep, or stress relaxation conditions as needed.

CREEP FAILURE CRITERION FOR MATERIALS ABS-A AND ABS-N

The creep results for ABS-A, at three different stresses (4138, 4635, and 5197 psi), have been included in Figure 3 and the creep results for ABS-N, at three different stresses (3227, 3703, and 4322 psi), have been included in Figure 4. The results in Figures 3 and 4 elucidate the different creep slopes in secondary creep and they also illustrate the common intercept strain ε_I, identified as the "projected elastic limit," which can be described by either eq. (22) or eq. (24). The calculated slopes in the secondary creep regions for these different stresses have been converted to extensional viscosities, as described elsewhere,[5,6] to allow the calculation of λ_E and n, as illustrated in Figure 5. The values for the extensional viscosity constant λ_E were evaluated primarily using eqs. (27) and (29).

Although the value of β can then be calculated directly from λ_E, as indicated by eq. (29), it has been found to be faster and simpler to calculate an average β to fit creep measurements using another technique. This second approach does require the calculation of the value for n, from creep extensional viscosity results plotted in Figure 5, using eq. (29). The strain, at critical creep ε_{CC}, was then calculated from eq. (21)

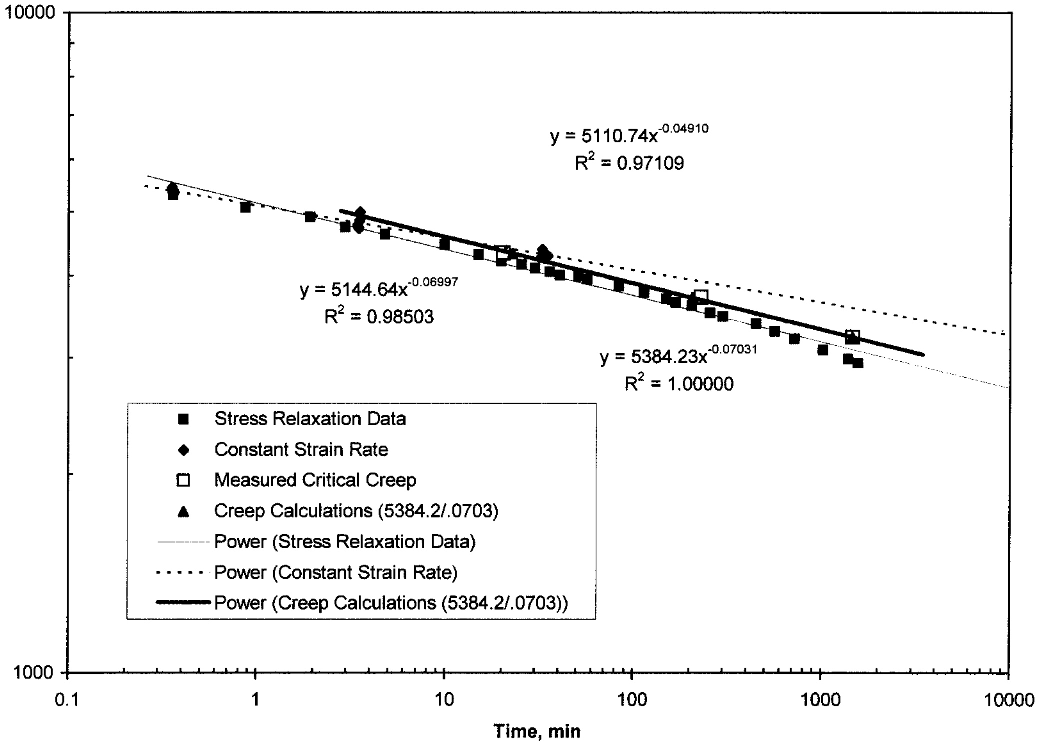

Figure 2. Constant strain rate, stress relaxation, and creep stress versus time for ABS material LL-4102-N.

using the value of n from creep measurements, and the constants for K and ε_∞ from constant strain rate measurements, as summarized in Table I. The time to reach critical creep t_{CC} was then calculated by substituting the strain at critical creep ε_{CC} into eq. (22), along with the straight line constants at each stress level

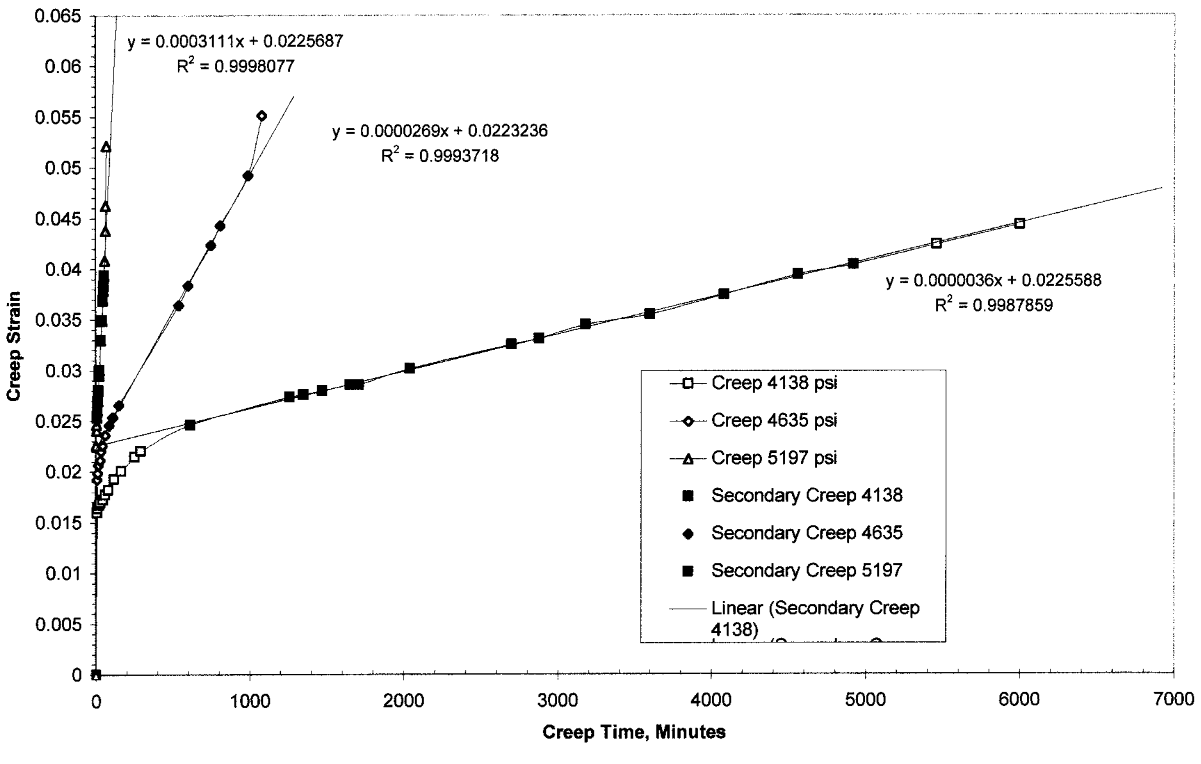

Figure 3. Creep strain versus creep time at three different stresses for ABS material 25383-A.

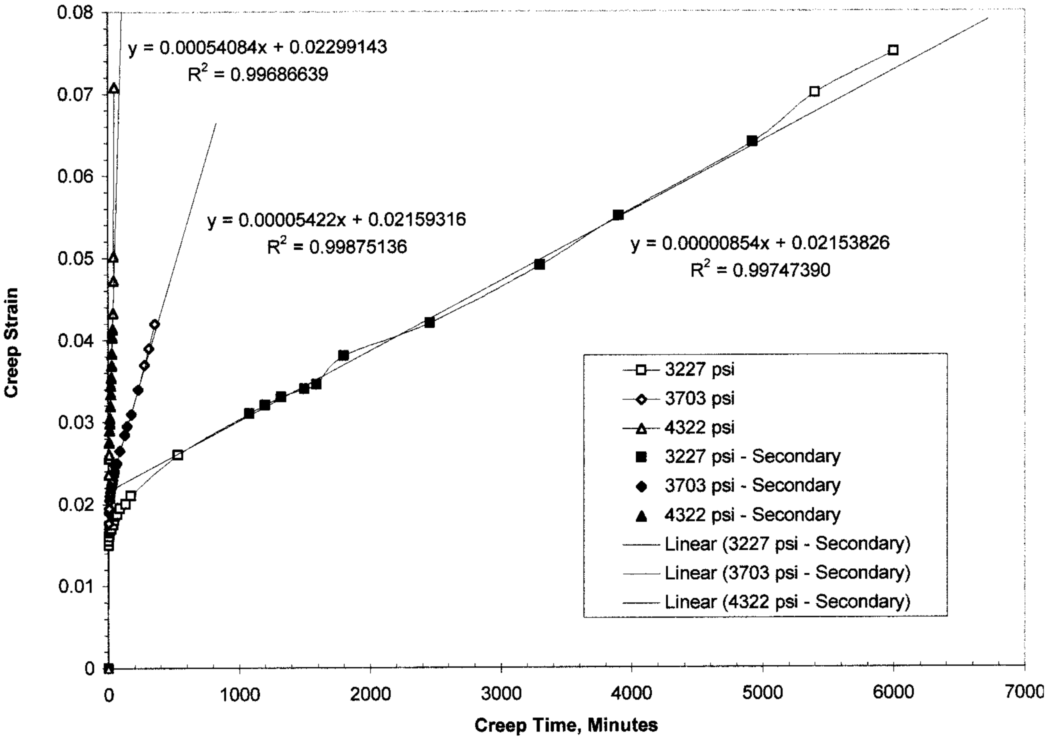

Figure 4. Creep strain versus creep time at three different stresses for ABS material LL-4102-N.

from Figures 3 and 4. The resulting values for the time to reach critical creep t_{CC} are indicated, in Figures 1 and 2, to be the failure conditions at each stress level.

As indicated in previous publications,[1,3,6] the condition at critical creep is considered to be essentially equivalent to the failure condition at the yield strain in

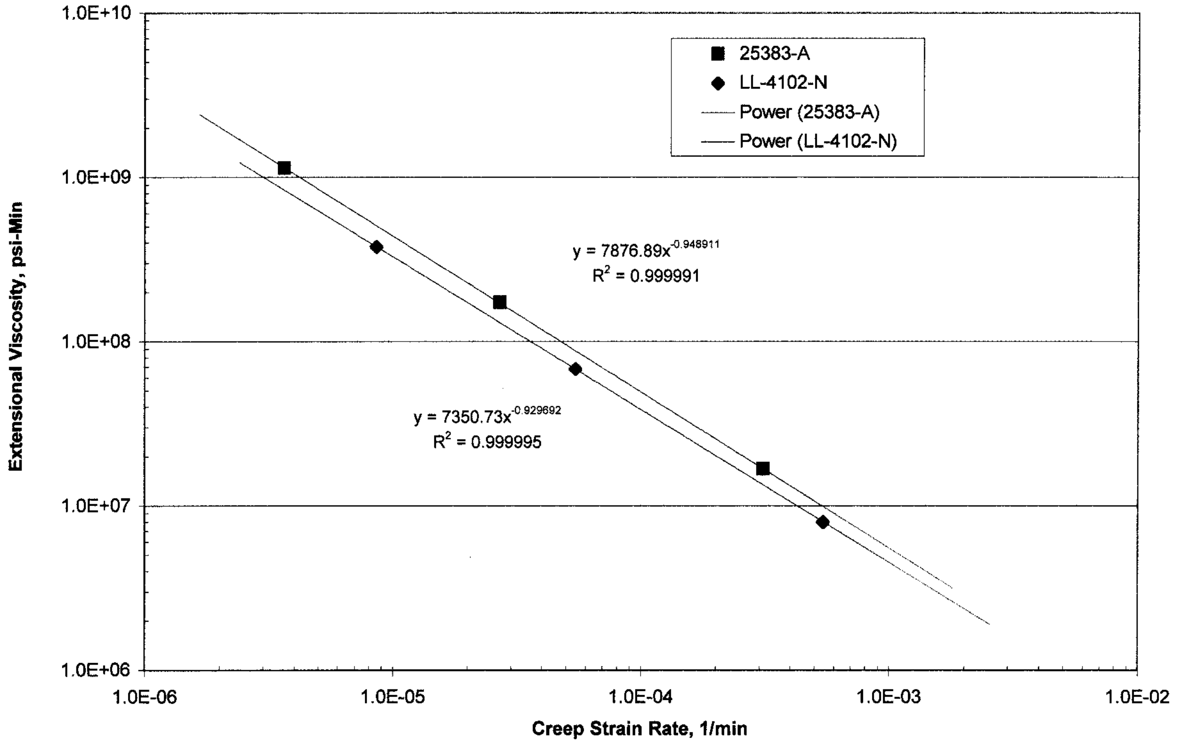

Figure 5. Extensional viscosity versus creep strain for two different ABS materials.

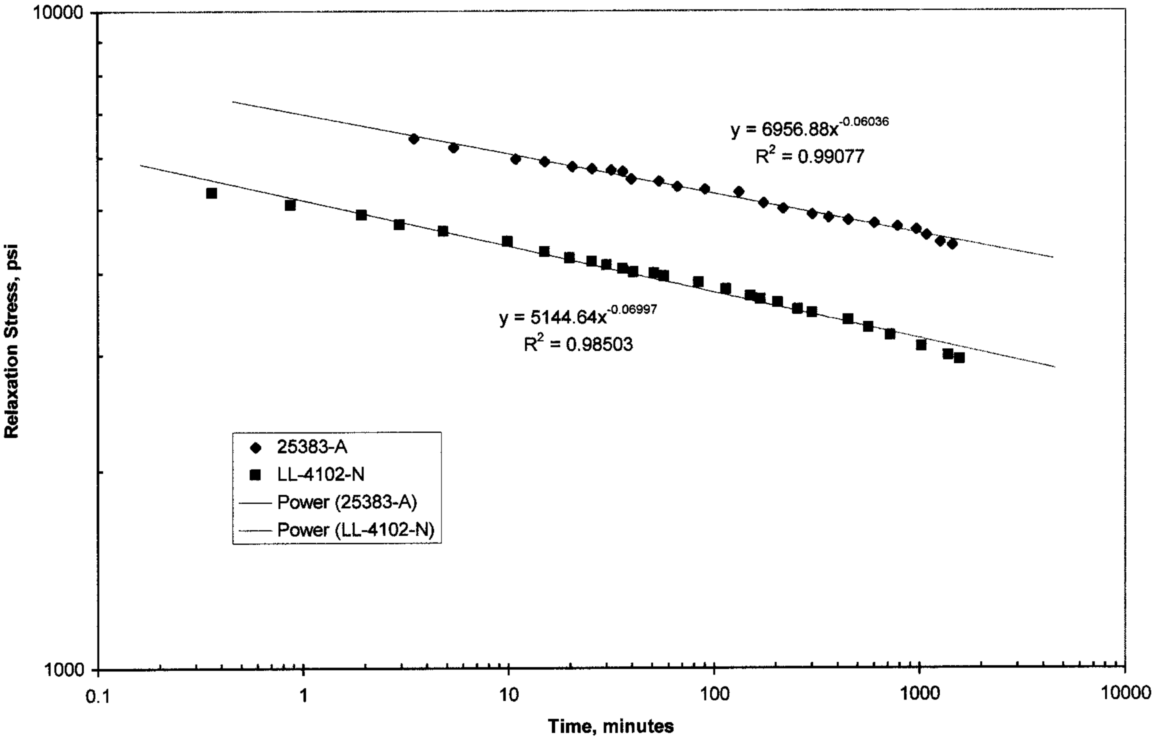

Figure 6. Stress relaxation versus time for ABS materials 25383-A and LL-4102-N.

a constant strain rate configuration using the universal viscoelastic model. The failure conditions for critical creep are summarized in Figures 1 and 2 for reference. The values of β for each creep stress level were then calculated from eq. (12), using the value for n from creep extensional viscosity measurements and the calculated value for the time to reach critical creep t_{CC}, as well as the known creep stress σ. The value of β that best fits the creep data was then obtained by averaging the calculated values of β calculated for each creep stress level. Although essentially this same value for β was also obtained using eq. (29), this second approach outlined here was found to be a much simpler calculation and it yielded a much better fit of the data.

A direct comparison of the creep constants for ABS-A and ABS-N is summarized in Table I. Notice that the values for n and β, from creep measurements for ABS-A in Table I, appear to be nearly identical to values for n and β from constant strain rate measurements. However, although the values of β were approximately the same for ABS-N for both creep and constant strain rate measurements, as indicated in Table I, the values for n for ABS-N appeared to be somewhat different.

In previous articles by this author,[2,3,6] as well as from similar discussions by Scott-Blair[35] and Hernandez-Jimenez,[36] it was found that the efficiency of yield energy dissipation n appears to range primarily from $0 < n < 1$. In this range a material would be characterized as being essentially purely elastic if $n = 0$, and essentially purely viscous or liquid in character if $n = 1$. Given that the efficiency of yield energy dissipation n, for ABS-N, as indicated in Table I, is slightly greater than the value of n for ABS-A, this would suggest that material ABS-A should have a slightly more solidlike character than material ABS-N. However, because the values of the efficiency of yield energy dissipation n, for both materials ABS-A and ABS-N, are so small, they would both be expected to be much more strongly solidlike than liquidlike. This is particularly important for failure conditions involving pipe burst for these materials at very long times.

STRESS RELAXATION FOR MATERIALS ABS-A AND ABS-N

The stress relaxations, at the yield stress for materials ABS-A and ABS-N, are plotted in Figure 6 at their respective yield strains as a function of time to generate values for n and β from a direct fit of this stress relaxation data to eq. (12). For reference, the stress relaxations, of both materials in Figure 6, were initiated using a constant strain rate of 2 in./min until the yield strain was achieved. The results in Figure 6 indicate that the stress relaxation locus of points for material ABS-A was greater than the level of stress relaxation for material ABS-N. Consequently, the β constant derived from stress relaxation measurements for ABS-A was found to be 35.2% higher than the value of β derived from stress relaxation for ABS-N. In

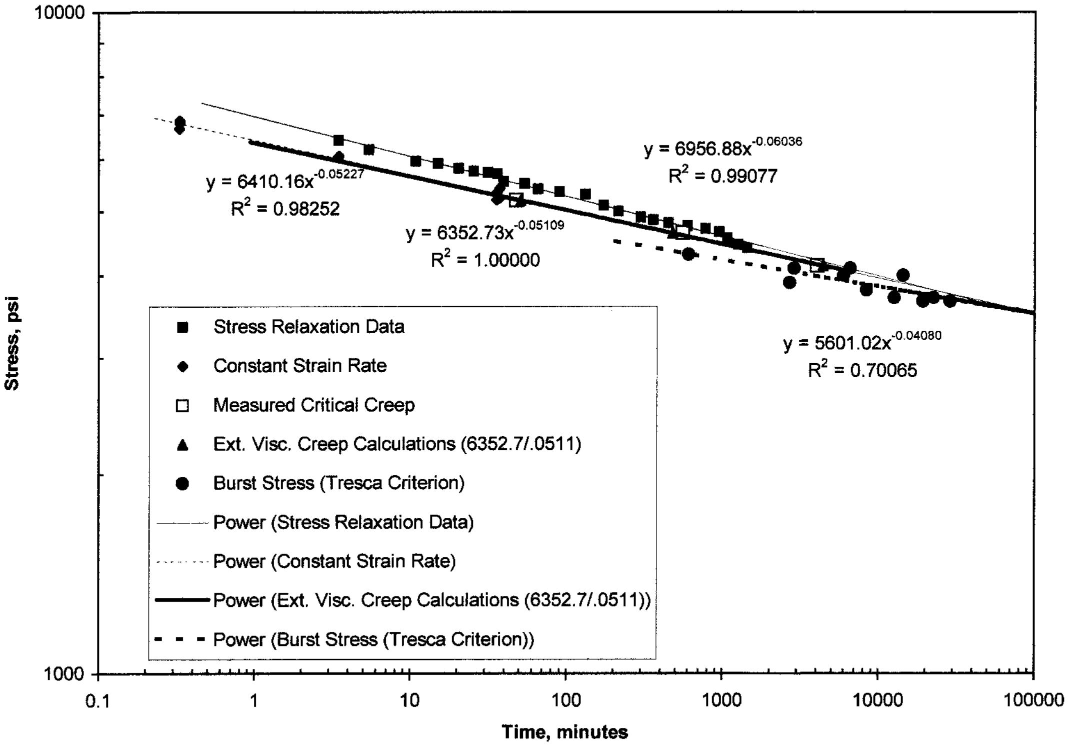

Figure 7. Constant strain rate, stress relaxation, and creep stress and pipe burst (Tresca criterion) versus time for ABS material 25383-A.

addition, the value for the efficiency of yield energy dissipation *n* for ABS-A was approximately 13.8% lower than the value of *n* for ABS-N. The stress relaxation constants for these two materials are also summarized in Table I. The stress relaxation values in Figure 6 are also included in Figures 1 and 2, so that a direct comparison can be made between the failure criteria for constant strain rate, creep, and stress relaxation.

COMPARISON OF PREDICTED FAILURE CONDITIONS FOR CREEP, CONSTANT STRAIN RATE, AND STRESS RELAXATION

If the yield strain is considered to be the failure condition for both constant strain rate and stress relaxation measurements and if the strain at critical creep is considered to be the failure condition for creep, then these failure conditions can be compared directly, as indicated in Figures 1 and 2 for materials ABS-A and ABS-N, respectively. The results indicated in Figures 1 and 2 were generated using the universal viscoelastic model addressed in this study. Some observations indicated in Figures 1 and 2 would include:

1. The failure criterion, summarized in both of these figures for the three different measurement techniques of constant strain rate, stress relaxation, and creep measurements, were in remarkably good agreement. This agreement resulted even though separate and independent measurements were used for these three different evaluation techniques for both materials ABS-A and ABS-N.
2. The extrapolation and overlap of the constant strain rate and creep measurements for material ABS-A in Figure 1 were extremely good. These results are even more remarkable because this agreement resulted from a comparison of separately measured results.
3. All of the failure criteria measurements in Figure 2 appear to merge together quite nicely. However, extrapolation to long-time failure conditions appears to give slightly different results for the three different techniques indicated.

COMPARISON OF THE PREDICTED FAILURE CONDITIONS FOR CREEP, CONSTANT STRAIN RATE, AND STRESS RELAXATION WITH ACTUAL PIPE BURST DATA

The pipe burst data for ABS material 25383-A are summarized in Figure 7, which also includes all the predicted failure conditions for constant strain rate, creep, and stress relaxation in Figure 1. Because the Tresca failure criterion, based on eq. (9), indicates that the yield stress is equal to the hoop stress in failure, then the burst stress in the hoop direction was compared directly with the yield stress equivalent failure

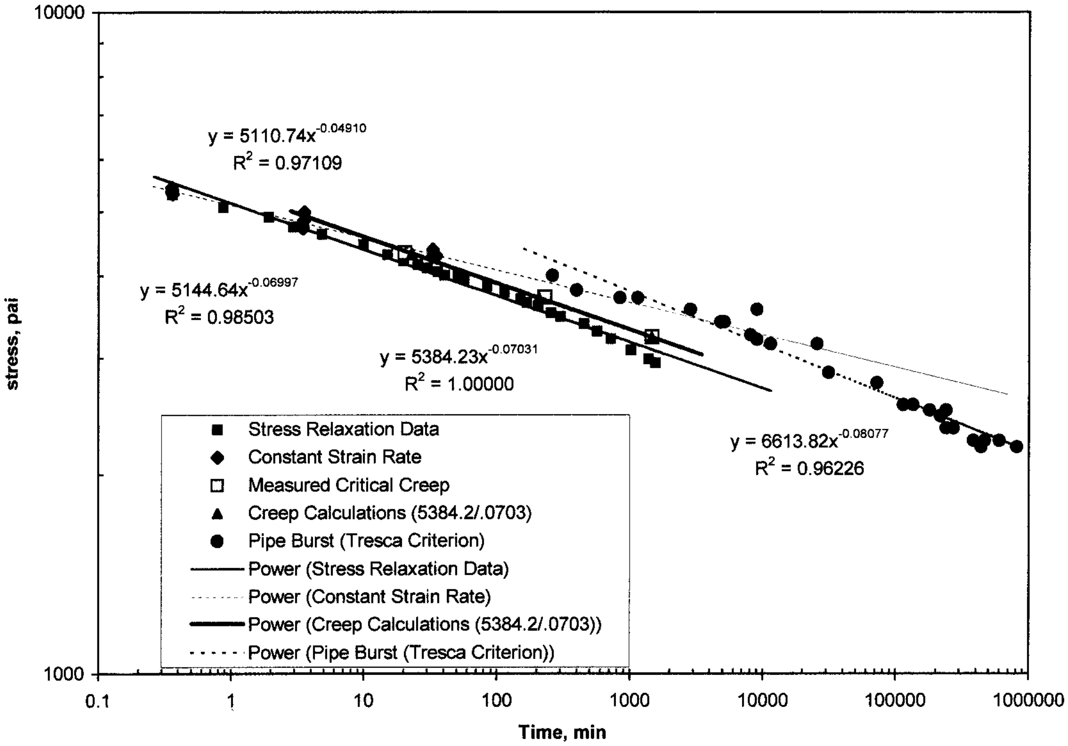

Figure 8. Constant strain rate, stress relaxation, and creep stress and pipe burst (Tresca criterion) versus time for ABS material LL-4102-N.

conditions for constant strain rate, creep, and stress relaxation in Figure 7. The results in Figure 7 appear to indicate that the creep data and the constant strain rate data best predict the pipe burst data for ABS material ABS-A. However, both the efficiency of yield energy dissipation n and the value for β were lower for the pipe burst measurements than for the creep and constant strain rate measurements of n and β, as indicated in Table I. Nevertheless, for ABS material ABS-A, there appeared to be a satisfactory agreement between the actual pipe burst measurements and their predicted values for the range of stress levels addressed in this study.

The pipe burst data for ABS material LL4102-N (ABS-N) are summarized in Figure 8, which also includes all the predicted failure conditions for constant strain rate, creep, and stress relaxation in Figure 2. Unfortunately, the Tresca failure criterion for the pipe burst failure conditions for ABS-N do not agree well with the yield stress equivalent failure conditions for constant strain rate, creep, and stress relaxation, as indicated in Figure 8. Consequently, the von Mises failure criterion described by eq. (4) was used to modify the hoop failure stress as follows:

$$\sigma_y = \left(\frac{\sqrt{3}}{2}\right)\sigma_H \tag{4}$$

As indicated in Figure 9, the hoop burst stress, adjusted with this von Mises failure criterion modification, was found to give better agreement with the yield stress equivalent failure conditions for constant strain rate, creep, and stress relaxation. The results, in Figure 9, appear to indicate that the creep data best predict the pipe burst data for ABS material ABS-N. However, both the efficiency of yield energy dissipation n and the value for β were higher for the pipe burst measurements than these same values for the creep measurements. Nevertheless, there again appeared to be satisfactory agreement between the actual pipe burst measurements and their predicted values, based on the stress relaxation measurements for the range of stress levels addressed in this study.

ADDITIONAL COMMENTS REGARDING THE INTERRELATIONSHIP BETWEEN PIPE BURST AND THE FAILURE CRITERION FOR CONSTANT STRAIN RATE, CREEP, AND STRESS RELAXATION

Although the strain to failure for the ABS pipe samples evaluated in this study were not measured, Malpass[10] did make such a direct comparison between the strain at burst and the yield strain for the ABS material evaluated in his study. What he found was that, by using approximately the same strain rate in the hoop direction as the uniaxial strain rate, the strain to burst in the hoop direction was almost identical to the strain to yield for uniaxial measurements. This result is extremely important because it lends further credence to

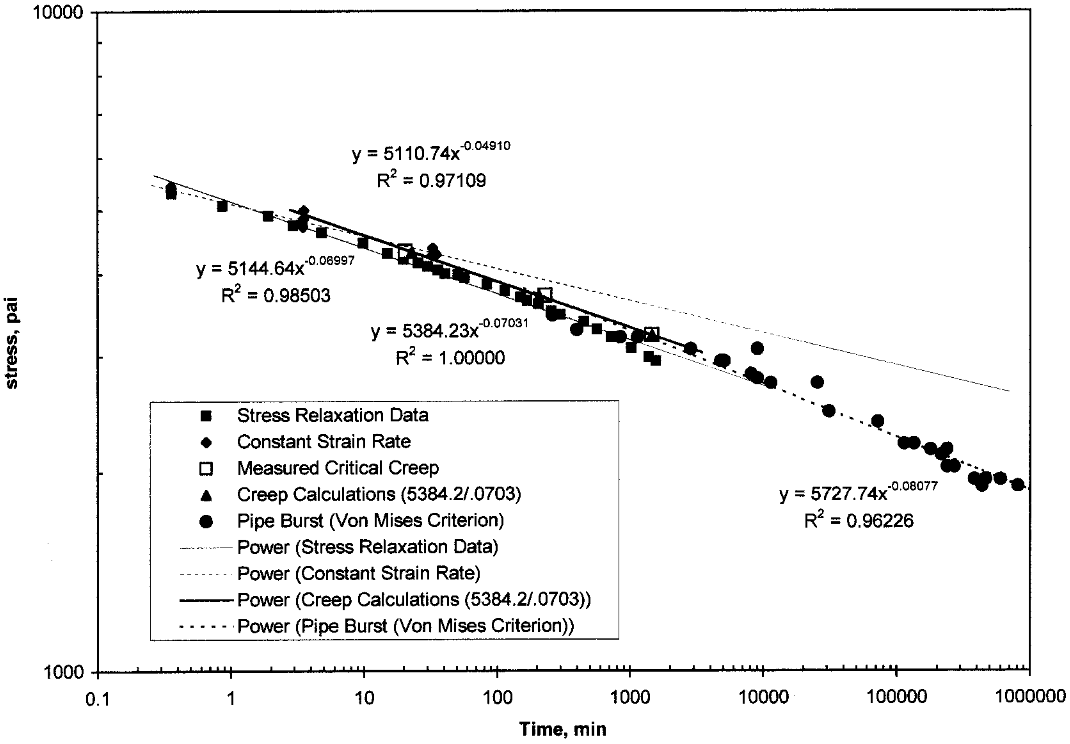

Figure 9. Constant strain rate, stress relaxation, and creep stress and pipe burst (von Mises criterion) versus time for ABS material LL-4102-N.

the theoretical observation that the strain to yield and the strain to critical creep, in the universal viscoelastic model, should be consistent with failure in pipe burst. Therefore, if stress relaxation is also evaluated at the yield strain, then all three methods of evaluation, including constant strain rate, creep, and stress relaxation, should give the same failure criterion. Although the results from this study do not necessarily show an exact relationship for the failure criteria between these three laboratory measurement techniques, the results do clearly show a remarkably close relationship between these three measurement approaches.

This interrelationship between these three different experimental techniques is often important when it may be desirable to predict the response from one measurement technique by using another technique that may be more available. For example, it is often desirable to be able to predict in the laboratory what stress needs to be set in the creep apparatus to get failure to occur within a specific time scale. Consistent with such a time scale it is often desirable to have an idea of how often to set time-measurement intervals to be able to evaluate all phases of the creep curve including primary, secondary, and tertiary creep. In general, it has been found that the constant strain rate measurements are usually the easiest, fastest, and most available instrumentation to predict all the constants needed to predict creep behavior at a specific stress level. It is often desirable then to follow constant strain rate measurements with stress relaxation measurements that can be used of confirm the constant strain rate measurement predictions. Finally, creep measurements can be used at the predicted stress level to confirm both the constant strain rate measurements and the stress relaxation measurements. It is also recognized, however, that creep measurements often require the longest time to obtain.

Because the above approach was found to work so well in the laboratory, there is no reason to believe that this same technique, using the universal viscoelastic model, would not also work as well to predict failure criteria for design applications. In particular, it would appear that this new universal viscoelastic model should be particularly useful for predicting failure conditions for applications involving finite-element analysis.

CONCLUSIONS

In general, the universal viscoelastic model, evaluated in this article, was found to adequately predict constant strain rate, creep, and/or stress relaxation measurements from the constants determined from constant strain rate measurements. The values for n and β, from creep measurements for ABS-A, appear to be nearly identical to values for n and β from constant strain rate measurements. However, although the values of β were approximately the same for ABS-N for both creep and constant strain rate measurements, the values for *n* for ABS-N appeared to be slightly different. Given that the efficiency of yield energy dissipation *n* for ABS-N was also slightly greater than the value of *n* for ABS-A, this would suggest that material ABS-A should have a slightly more solidlike character than material ABS-N. However, both materials ABS-A and ABS-N were found to be generally more solidlike than liquidlike, given that the values of their efficiency of yield energy dissipation *n* were both so low. This was found to be particularly important for failure conditions using these materials at very long times.

In general, the yield strain was considered to be the failure condition for constant strain rate and stress relaxation measurements and the strain at critical creep was found to be a primary failure condition for creep. These observed failure criteria for constant strain rate, stress relaxation, and creep measurements were found to yield remarkably good agreement using the universal viscoelastic model in this study. In particular, the extrapolation and overlap of the constant strain rate and creep measurements for material ABS-A were particularly notable. This agreement resulted even though separate and independent data were used to evaluate these three different techniques for both materials ABS-A and ABS-N.

The pipe burst data for ABS material ABS-A appeared to fit the Tresca failure criterion better than the von Mises failure criterion, compared to the predicted failure criteria from constant strain rate, creep, and stress relaxation. For ABS material ABS-A there appeared to be a satisfactory agreement between the actual pipe burst measurements and their predicted values using all three failure criteria involving constant strain rate, creep, and stress relaxation for the range of stress levels addressed in this study. However, for ABS-A the extrapolated creep and the constant strain rate failure criteria appeared to best predict the Tresca pipe burst failure data.

Unfortunately, the Tresca failure criterion for the pipe burst failure conditions for ABS-N did not agree well with the yield stress equivalent failure conditions for constant strain rate, creep, and stress relaxation. The hoop burst stress adjusted with the von Mises failure criterion modification was found to give better agreement with the yield stress equivalent failure criterion for constant strain rate, creep, and stress relaxation. Again, for ABS-N very satisfactory agreement was obtained between the actual pipe burst measurements and their predicted values using all three failure criteria involving constant strain rate, creep, and stress relaxation for the range of stress levels addressed in this study. However, for ABS material ABS-N the extrapolated creep failure criteria appeared to best predict the von Mises pipe burst failure criterion.

Although the strain to failure was not evaluated for the ABS pipe samples evaluated in this study, Malpass[10] found that, for ABS materials, the strain to burst in the hoop direction was almost identical to the strain to yield for uniaxial measurements. This result was found to be extremely important because it indicated that the strain to yield for constant strain rate, the strain to critical creep, and the stress relaxation evaluated at the yield strain should all give the same failure criterion. Although the results from this study do not necessarily show an exact relationship for the failure criteria between these three laboratory measurement techniques, the results do clearly show a remarkably close relationship between these three measurement approaches.

In this study, the relationships between the failure criteria, for the three different experimental techniques of constant strain rate, creep, and stress relaxation, have been shown to be reasonably interchangeable with respect to a three-dimensional failure configuration such as pipe burst. This is often very important when it may be desirable to predict the failure criterion response from one configuration by using data from another failure criterion configuration that may be more available. In general, the interchangeability of the prediction of nearly identical failure criterion using the universal viscoelastic model was found to work extremely well in the laboratory. As a result, there is every reason to believe that these same techniques would also work equally well to predict failure criterion for design applications using finite-element analysis.

The author acknowledges the GE Corporation for allowing publication of the constant strain rate, creep and stress relaxation, and pipe burst measurements for the two ABS materials 25383-A (ABS-A) and LL-4102-N (ABS-N) presented in this study. These measurements were generated by this author at GE's Washington, WV subsidiary.

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